

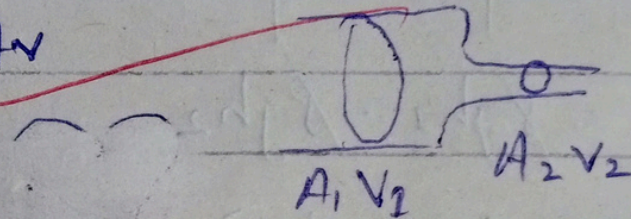
# Fluid Dynamics

## Continuity:

→ Flow rate must be conserved.  
(conservation of mass)

Flow rate:

$$Q = Av$$

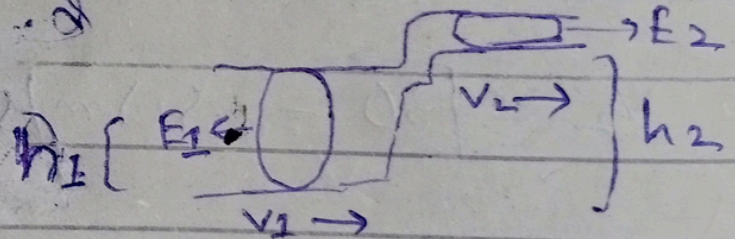


$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

## Bernoulli's Principle:

→ Energy is conserved.



$$E_{\text{internal}} + \frac{1}{2}mv^2 + mgh$$

Dividing by volume:

$$P + \frac{1}{2}\rho v^2 + \rho gh$$

$$\rho = \frac{m}{V}$$

## Bernoulli's Equation:

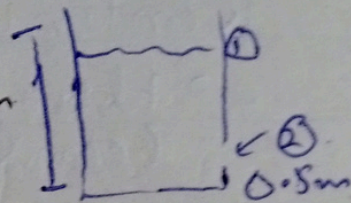
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

A tank of water 2m tall has an open top and an opening 0.5m from the base. What is the velocity of fluid escaping?

*velocity is very slow*

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad 2m$$

↳ both are atmospheric pressure.



$$\frac{1}{2} \rho v_2^2 = \rho g h_1 - \rho g h_2$$

$$v_2^2 = 2g(h_1 - h_2)$$

$$v_2^2 = 2(10)(2 - 0.5)$$

$$v_2^2 = 20(1.5)$$

$$v_2^2 = 30$$

$$v_2 = \sqrt{30} = 5.5 \text{ m/s}$$

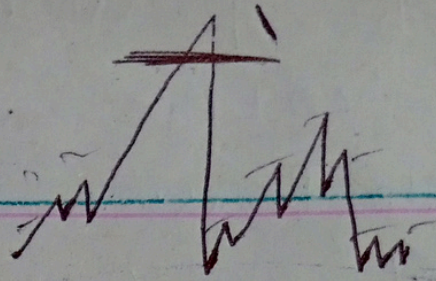
$$\begin{array}{r} 20 \\ \times 1.5 \\ \hline 100 \\ 200 \\ \hline 300 \end{array}$$

Venturi Effect:

→ higher velocity means lower pressure in a given system.

$$P_2 = P_1 - \frac{1}{2} \rho (v_2^2 - v_1^2)$$

1/6



Density:

Density: how much mass per unit volume.

$$\rho = \frac{m}{V}$$

Specific gravity: density relative to water

$$= \frac{\rho_{\text{substance}}}{\rho_{\text{water}}}$$

→ gases are fluids!

Pressure:

→ force per unit area.

$$P = \frac{F_{\perp}}{A}$$

$$[P] = \frac{\text{kg}}{\text{m s}^2} = \text{Pascals (Pa)}$$

$$\text{Pascals} = \frac{\text{N}}{\text{m}^2} = \frac{\text{J}}{\text{m}^3}$$

Force perpendicular to wall.

Atmospheric pressure:

$$1 \text{ atm} = 100 \text{ kPa}$$

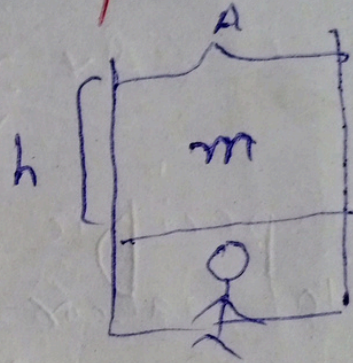
Hydrostatic Pressure:

$$P = \rho g h$$

↳ Gauge Pressure: amount by which pressure measured in a fluid exceeds atmospheric pressure:

$$P_{\text{gauge}} = \rho g h$$

$$P_{\text{total}} = \rho g h + P_{\text{atm}}$$



Archimedes Principle:

Buoyancy:

$$F_B = (m_{\text{displaced water}}) g$$

An object will float if:

$$\rightarrow F_B > F_g$$

$$\rightarrow m_{\text{water}} g > m_{\text{object}} g$$

$$\rightarrow \rho_{\text{water}} V > \rho_{\text{object}} V$$

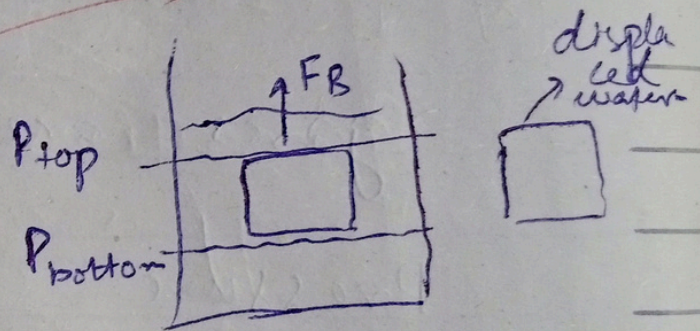
$$\rightarrow \rho_{\text{water}} > \rho_{\text{object}}$$

if an object is floating, it will emerge from water until:

$$\rightarrow F_B = F_g$$

$$\rightarrow m_{\text{water displaced}} = m_{\text{object}}$$

$$\rightarrow \rho_{\text{water}} V_{\text{submerged}} = \rho_{\text{object}} V_{\text{object}}$$



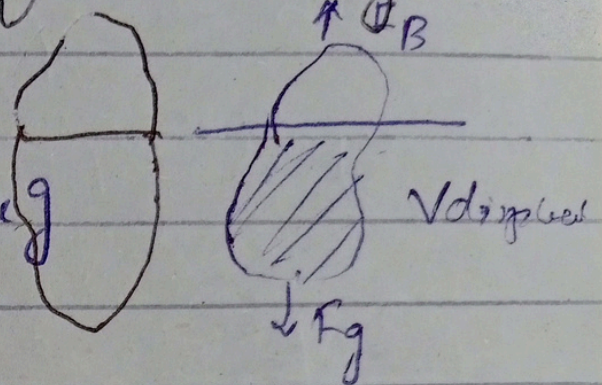
↳ Force  $F_B$  upwards is equal to the weight of the displaced water.

For Floating Object

$$\frac{V_{\text{submerged}}}{V_{\text{object}}} = \frac{\rho_{\text{object}}}{\rho_{\text{water}}}$$

The specific gravity of ice is approximately 0.9. What fraction of an iceberg is really underwater?

$$F_B = mg = (\rho V)g = F_g = m_{ice}g$$



$$\rho_w V_w g = \rho_i V_i g$$

$$\frac{\rho_w}{\rho_i} = \frac{V_i}{V_w} \quad \text{or} \quad \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = 0.9$$

Specific gravity = 0.9  
90% → under water

Pascal's Law:

→ Pressure is equally distributed throughout a fluid.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Equal force per unit area.

Hydraulics:

→ you gain a mechanical advantage by making one area greater than another.

$$F_2 = F_1 \frac{A_2}{A_1}$$