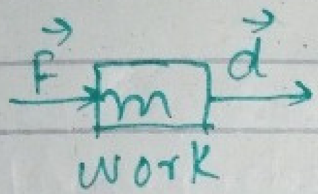
02:

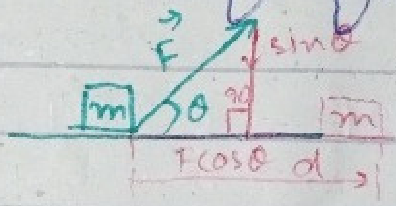
Work & Energy

Work Done by Cons. Force:

$\vec{F} = F\hat{F}$, when both magnitude & direction are same the F will be constant.



When force is applied on body and it covers displacement in direction of force.



$W = (F_x)(d)$

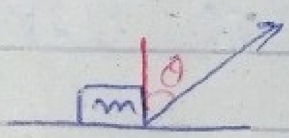
$W = F\cos\theta . d$

$W = \vec{F} . \vec{d}$

→ Dot product of $\vec{F}$ and $\vec{d}$.

→ Scalar Quantity

→ No direction.



$\cos\theta \to \sin\theta$

$\sin\theta \to \cos\theta$

$W = Fd\sin\theta \Rightarrow \vec{F} . \vec{d}$

→ θ along y-axis

Dependance:

$W = Fd\cos\theta$

* F, d, θ b/w $\vec{F}$ and $\vec{d}$

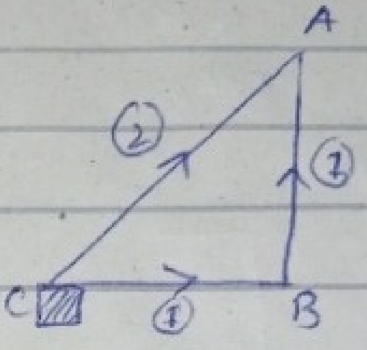
$W = Fd\cos\theta$

0°	30°	45°	60°	90°	180°
$W_{max} = Fd$,	$\frac{\sqrt{3}}{2} Fd$	$\frac{1}{\sqrt{2}} Fd$	$\frac{1}{2} Fd$	0	$-Fd$
W_{max},	$\frac{\sqrt{3}}{2} W_{max}$	$\frac{1}{\sqrt{2}} W_{max}$	$\frac{1}{2} W_{max}$	0	$-W_{max}$
100%	86%	70%	50%		

Work Done by Variable Force:

$\vec{F} = F\hat{F}$ → When magnitude or direction or both changes.

→ Conservative Field:

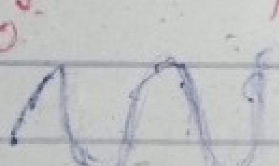


C → A

$W_{C-B-A} = -mgh$ $W_{C\to A} = -mgh$

→ Work done is independent of path ~~forward~~ followed.

→ Work along closed path will be zero



WOF represents co
vative field

a: W = 0 (For closed

b: W depends upon pat fo

c: W independent path fo

d: a & c

• Example:

→ Gravitational field → Electric field → Elastic spr

→ Non-Conservative field:

→ Work depends upon path followed.

Example:

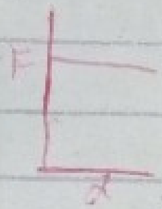
- Frictional force → mcq
- Tension in string
- Rocket propulsion

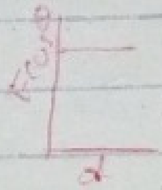
Work Done Graphically:

→ By constant Force:

$\vec{F} \parallel \vec{d}$ $\quad$ $\vec{F} \nparallel \vec{d}$

$\theta = 0°$ $\quad$ $\theta \neq 0°$





slope $= \frac{F}{d} = \frac{N}{m}$

$F = kx \Rightarrow k = \frac{F}{x} = \frac{N}{m}$

slope = spring constant

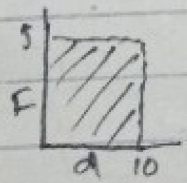
or

slope = surface tension $= \frac{F}{L}$

Area = (Y axis) (x axis)

$= F \cdot d$

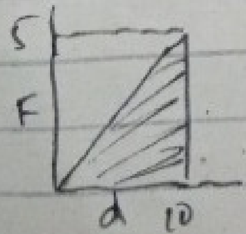
Area = W



Area = ?

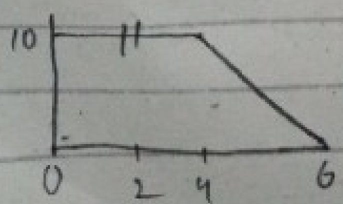
$A = F \times d$

$A = 5 \times 10 = 50J$



$A = \frac{1}{2}(L \times W)$

$A = \frac{1}{2}(10 \times 5) = 25J.$

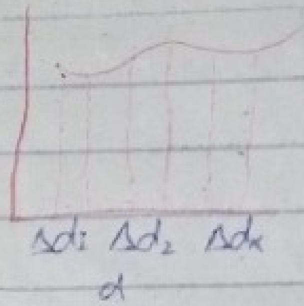


$A = \square + \triangle$

$= 4 \times 10 + \frac{1}{2}(2 \times 10)$

$= 40 + 10 = 50J.$

→ By Variable Force:



→ For more accurate work. $\boxed{\Delta d \to 0}$

$W = \sum_{i=1}^{n} \left(\lim_{\Delta d \to 0} F_i \cos\theta_i \, \Delta d_i \right)$

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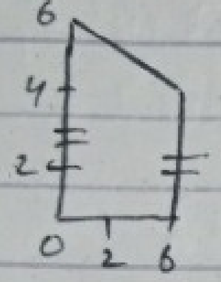
Trapezium:

$$A = \frac{1}{2}(\text{sum of } \parallel \text{ sides}) \times \perp \text{ side}$$

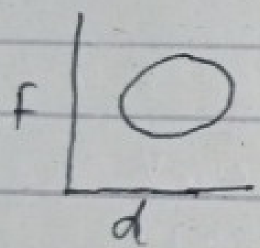
$$= \frac{1}{2}(6+4) \times 10$$

$$A = \frac{1}{2}(10) \times 10 = \frac{100}{2} = 50 J$$



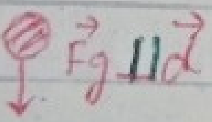
$$A = \frac{1}{2}(6+4) \times 6)$$

$$A = \frac{1}{\cancel{2}}(60) = 30$$

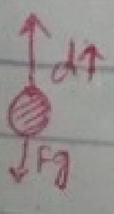


$W = F(0) = 0 J$ * displacement of circle is zero

Work Done by Gravity.



W = +ive
P.E = decrease
K.E = increase



W = -ive
P.E = increase
K.E = decrease

Energy:

→ Ability to do work.

$J = Kg\,m^2 s^{-2}$

Mechanical Energy

K.E:

* Ⓔ due to motion

* $K.E = \frac{1}{2} m v^2$

Its speed not velocity

P.E

* Ⓔ due to position

for larger dist $g \neq 10$

near surface of earth

* $P.E = mgh$

$P.E \propto h$

$A.G.P.E \propto R$

* $A.G.P.E = -\frac{GMm}{R}$

* $E.P.E = \frac{1}{2} K x^2$.

$K.E = \frac{1}{2} m (\vec{v} \cdot \vec{v})$

$K.E = \frac{1}{2} m v^2$

$= \frac{1}{2} m v v$

$\boxed{K.E = \frac{1}{2} P v}$

$K.E = \frac{1}{2} m v^2 \times \frac{m}{m}$

$K.E = \frac{m^2 v^2}{2m}$

$\boxed{K.E = \frac{P^2}{2m}}$

$P^2 = 2m K.E$

$P = \sqrt{2m K.E}$

$K.E = \frac{P^2}{2m}$

• P = constant

$K.E \propto \frac{1}{m}$

$\frac{K.E_1}{K.E_2} = \frac{m_2}{m_1}$

mcq: If P = const ratio of masses 1:4. ratio of K.E?

4 : 1

• m = constant

$K.E \propto P^2$

$\frac{K.E_1}{K.E_2} = \frac{P_1^2}{P_2^2}$

mcq If m = constant If ratio of momen is 2:7, K.E = ?

4 : 49

If % ↑P = 20%.

then % ↑ K.E = ?

% ↓↑ P = given

% ↓↑ K.E = 2 (% P) + n^2

n: 1st digit after decimal. It may be zero.

0.2 → n

0.03 → n

$20\% = \frac{2\cancel{0}}{10\cancel{0}} = 0.2 \rightarrow n$

$\%\uparrow K.E = 2(20\%) + (2)^2$

$\boxed{\%\uparrow K.E = 44\%}$

If $\%\uparrow P = 3\%$, $\% K.E = ?$

$3\% = \frac{3}{100} = 0.03 \rightarrow n = 0$

$\%\uparrow K.E = 2(\%3) + 0^2$

$\boxed{\%\uparrow K.E = 6}$

Power:

Rate of doing work

$P = \frac{W}{t} = \frac{E}{t} = \frac{K.E}{t} = \frac{P.E}{t}$

$P = \frac{Fd}{t}, \frac{F}{t} = \frac{mv^2}{2t}, \frac{mgh}{t}$

$P = \frac{Fvt}{t} = \boxed{P = Fv}$

$P = \vec{F}\cdot\vec{v}$

$P = Fv\cos\theta$

Wg Wg

MCQ:

If a motor pump takes 10 kg water from earth and fill in tanki of the heig 50m in 10s. What is power.

$P = \frac{\cancel{10}\times 10\times 50}{\cancel{10}} = 500\ J/s \rightarrow \text{watt}$

Unit:

$P = \frac{W}{t} = \frac{J}{s} = \frac{Nm}{s}$

$P = Nms^{-1} = Js^{-1} = \text{Watt}$

$1hp = 746\ \text{watt}$

$2hp = 2\times 746\ \text{watt}$

$= 1492\ \text{watt}$

Efficiency: (eta = η)

$\eta = \frac{P_o}{P_i} = \frac{E_o}{E_i} = \frac{W_o}{W_i}$

$\boxed{\%\eta = \frac{P_o}{P_i}\times 100\%}$

mcq: $P_o = 250W$

$\%\eta = 25\%$, $P_i = ?$

$\eta = \frac{25}{100}$ $\quad \eta = \frac{P_o}{P_i}$

$P_i = \frac{250}{\frac{25\cancel{0}}{100}} = \frac{250}{25\cancel{0}}\times 100$

Work Energy Principle

$P_i = \frac{288000}{281}$ ×100

No Air Friction:

Loss P.E = Gain in K.E

$mgh = \frac{1}{2}mv^2$

$2gh = v^2$

$v = \sqrt{2gh}$

With air Friction

↓ Loss P.E = Gain K.E + W (against air friction)

$100J = 90J + 10J$

$mgh = \frac{1}{2}mv^2 + Fh$

↑ Loss K.E = Gain P.E + W (against air friction)

$100J = 70J + 30J$

$\frac{1}{2}mv^2 = mgh + Fh$

$\vec{F} = 2i + 3j$

$\vec{d} = 3i + 4j$ W = ?

$W = \vec{F} \cdot \vec{d}$

$= (2i + 3j) \cdot (3i + 4j)$

$= 6 + 12 = 18J$

$\vec{F} = 2i + 3j + 4k$

$\vec{d} = 2i + 3j + 0k$

$W = 4 + 9 + 0$

$W = 13J$

@isamiqamar

$F = 2i + 3j$

· 3m → along x-axis

$d = 3i + 0j + 0K$

$F = 2i + 3j$

$W = 2 \times 3$

$= 6J$

Unit of θ:

SI → Radian

Non-SI → Degree
→ Revolution

How many degree in
(a) 57.3° b: 53.7°

$1\,rad = 57.3^\circ$

$1\,rad = 57.3 \times 1^\circ$

$$\boxed{1^\circ = \frac{1}{57.3} = 0.0174\,rad}$$

$1\,rev = 2\pi\,rad = 360^\circ$

$1\,rev = 2\pi\,rad$, $2\pi\,rad = 360^\circ$

$1\,rev = 2\pi \times 1\,rad$, $1\,rad = \frac{\overset{180}{\cancel{360}}^\circ}{\cancel{2}\pi}$

$1\,rad = \frac{1\,rev}{2\pi}$

$1\,rad = \frac{180^\circ}{\pi}$

$1\,rad = 0.159\,rev$, $\boxed{1\,rad = 57.3^\circ} \approxeq 60^\circ$

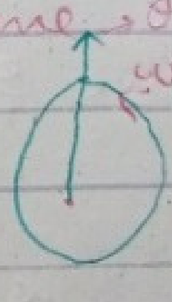
Angular Velocity:

→ Rate of change of angular displacement

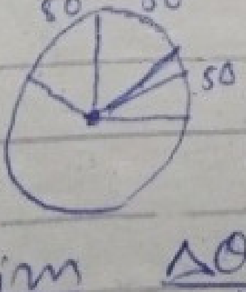
$\vec{\omega} = \frac{\Delta \vec{\theta}}{\Delta t}$ $\left(\frac{rad}{s}, \frac{rev}{min}\right)$

→ along normal to plane of θ

→ $\perp$ to plane



Uniform:

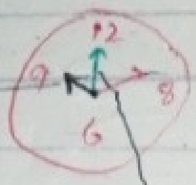


$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$

$\omega_{ins} = \omega_{av}$ → body is moving with uniform angular velocity.

$\omega_{av} = \frac{\Delta \theta}{}$

Time Clock System:



Minute Hand

$\theta = 2\pi\, rad$

$t = 60\, min$

$t = 3600\, s$

$\omega_m = \frac{2\pi}{60} \frac{rad}{min}$

$\omega_{M.H} = \frac{\pi}{30}\, rad/min$

$\omega_{M.H} = \frac{\pi}{1800}\, rad/s$

Hour Hand

$\theta = 2\pi\, rad$

$t = 12h$

$t = 43{,}200\, s$

$\omega_{H.H} = \frac{2\pi}{12}\, rad/h$

$\omega_{H.H} = \frac{\pi}{6} \frac{rad}{h}$

$\omega_{H.H} = \frac{2\pi}{43200}$

$\omega_{H.H} = \frac{\pi}{21600}\, rad/s$

second hand.

$\theta = 2\pi\, rad$

$t = 60\, s$

$t = 1\, min$

$\omega_{S.H} = \frac{2\pi}{60} \frac{rad}{s}$

$\omega_{S.H} = \frac{\pi}{30} \frac{rad}{s}$

Angular Acceleration:

→ Rate of change of angular velocity.

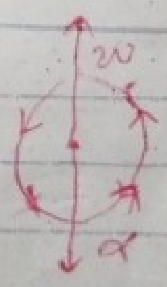
$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} \left(\frac{rad\, s^{-1}}{s}\right)$

→ $v\uparrow$, θ b/w $\vec{v}$ & $\vec{\alpha} = 0°$

→ $v\downarrow$, θ b/w $\vec{v}$ & $\vec{\alpha} = 180°$

→ $\omega\uparrow$, angle b/w $\vec{\omega}$ & $\vec{\alpha} = 0$

→ $\omega\downarrow$, angle b/w ω & $\vec{\alpha} = 180°$



$d \rightarrow \theta$

$v \rightarrow \omega$

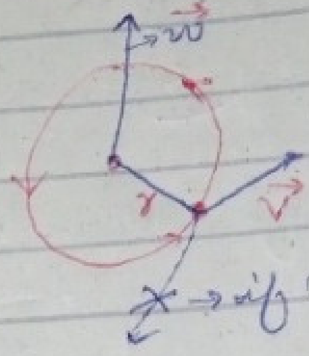
$a \rightarrow \alpha$

$P \rightarrow L$

$\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Relatio b/w Linear & Angular Quantities:



$S = r\theta$

$\vec{S} = \vec{\theta} \times \vec{r}$

$V = rw$

$\vec{V} = \vec{w} \times \vec{r}$ $\quad \vec{v} \neq \vec{r} \times \vec{w}$

$\times \rightarrow$ if r is first vector.

$\therefore \vec{w}, \vec{v}$ & $\vec{r}$ always $\perp$ to each other.

$a = r\alpha$

$\vec{v} = \vec{w} \times \vec{r}$

$\vec{a} = \vec{\alpha} \times \vec{r}$

$v \perp w$

$v \perp r$

$r \perp v, w$

Tangential Velocity & Acceleration

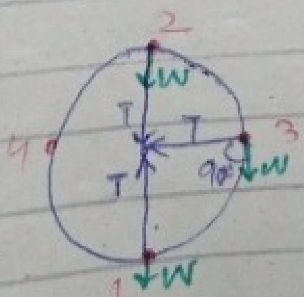
$\Delta t \longrightarrow 0$

$v \longrightarrow v_t$ $\quad V_t = rw$

$a \longrightarrow a_t$ $\quad a_t = r\alpha$

Circle:

1: Vertical Circle:



If body moves in a vertical circle the maximum chance to break the strength at:

a: top (b:) bottom

d: middle

$T = \frac{mv^2}{r} - W\cos\theta$

"θ" is angle b/w "W" & "T"

$V = \sqrt{3gr - 2gr\cos\theta}$

Bottom(1) : $\theta = 180°$

$$T_B = \frac{mv^2}{r} + W$$

$T_B > T_m > T_T$

Top(2) : $\theta = 0°$

$$T_T = \frac{mv^2}{r} - W$$

Mid (3, 4): $\theta = 90°$

$$T_m = \frac{mv^2}{r}$$

Centripetal Force:

Other Name:

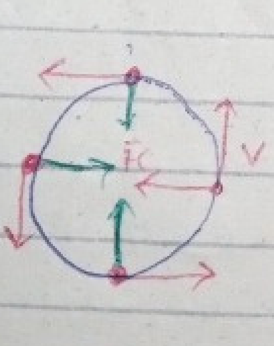
→ Center Seeking Force

→ Tendency to overturn

∴ The force which compels the body into circular path:

$$F = \frac{mv^2}{r}$$

∴ In circle speed will be same but velocity varies



v = constant

If: m = constant

$$F \propto \frac{1}{r}$$

$v = r\omega$

$$F = \frac{m(r\omega)^2}{r}$$

$$= \frac{mr^2\omega^2}{r} \quad \boxed{F_c = mr\omega^2}$$

If: ω = constant

$F \propto r$

Acceleration (a_c):

$F_c = ma_c$

$\frac{\cancel{m}v^2}{r} = \cancel{m}a_c$

$a_c = \frac{v^2}{r}$, $\boxed{a \propto \frac{1}{r}}$

$\because v = r\omega$

$a_c = \frac{r^{\cancel{2}}\omega^2}{\cancel{r}}$

$a_c = r\omega^2$ $\boxed{a \propto r}$

@isamiqamar

A body moves in circle of radius 2m with speed 4m/s. a = ?

$a_c = \frac{v^2}{r} = \frac{4^2}{2} = \frac{16}{2}$

$a_c = 8 m/s^2$

A body moves in circle of radius 2m with speed 4 rad s⁻¹ , a = ?

$a_c = r\omega^2$

$a_c = 2(4)^2 = 2(16)$

$a_c = 32 m/s^2$

∴ In both m.c.q) the unit of acceleration will be m/s^2 b/c we are finding linear acce

Relation b/w F_c, K.E & P:

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{mv^2}{r} \times \frac{m}{m}$$

$$F_c = \frac{m^2v^2}{rm}$$

$$F_c = \frac{p^2}{rm}$$

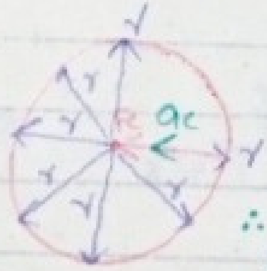
$$F_c = \frac{mv^2}{r} \times \frac{2}{2}$$

$$F_c = \frac{mv^2}{2}\left(\frac{2}{r}\right)$$

$$F_c = \frac{2K.E}{r}$$

Vector Form:

$$\vec{F_c} = -\frac{mv^2}{r}\hat{r}$$



Both Fc and ac will act in same direction.

∴ But the angle b/w {Fc, ac} and r will be 180°, that's why −ve sign.

$\because \vec{A} = A\hat{A}$

$\because \hat{A} = \frac{\vec{A}}{A}$

$\therefore \vec{r} = r\hat{r}$

$\hat{r} = \frac{\vec{r}}{r}$

$$\vec{F_c} = -\frac{mv^2}{r}\left(\frac{\vec{r}}{r}\right)$$

$$\vec{F_c} = -\frac{mv^2}{r^2}\vec{r}$$

Q) $m = 10kg$, $v = 2m/s$

$\vec{r} = 3i + 4j$

$r = \sqrt{3^2 + 4^2}$

$r = 5$

$$\vec{F_c} = -\frac{mv^2}{r^2}\vec{r}$$

$$\vec{F_c} = \frac{-(10)(2)^2}{5^2}(3i+4j)$$

$$\vec{F_c} = -\frac{8}{[illegible]}(3i+4j)$$

$$\vec{a_c} = \frac{-v^2}{r}\hat{r}$$

$$\vec{a_c} = \frac{-v^2}{r}\left(\frac{\vec{r}}{r}\right)$$

$$\vec{a_c} = \frac{-v^2}{r^2}\vec{r}$$

$$a_c = r\omega^2$$

$$\vec{a_c} = -r\omega^2\hat{r}$$

$$\vec{a_c} = -\omega^2\vec{r}$$

$$\vec{F} = -mr\omega^2\hat{r}$$

$$\because \hat{r} = \frac{\vec{r}}{r}$$

$$\vec{F_c} = -mr\omega^2\left(\frac{\vec{r}}{\cancel{r}}\right)$$

$$\vec{F_c} = -m\omega^2\vec{r}$$

Circular Motion:

Uniform ($a_c \neq 0$, $a_t = \alpha = 0$)

- $\to \omega = \text{constant}$
- $\to \Delta\omega = 0$
- $\to \alpha = \frac{\Delta\omega}{\Delta t}$
- $\to \boxed{\alpha = 0}$
- $\to a_t = r\alpha$
- $\to \boxed{a_t = 0}$
- $\to \boxed{a_c \neq 0}$

Variable ($a_c \neq 0$, $a_t \neq 0$, $\alpha \neq 0$)

- $\to \omega \neq \text{constant}$
- $\to \Delta\omega \neq 0$
- $\to \alpha \neq 0$
- $\to a_t \neq 0$
- $\to a_c \neq 0$