# 8: Waves

→ Disturbance in medium.
→ Transfer energy & momentum
→ Donot transfer Matter/mass

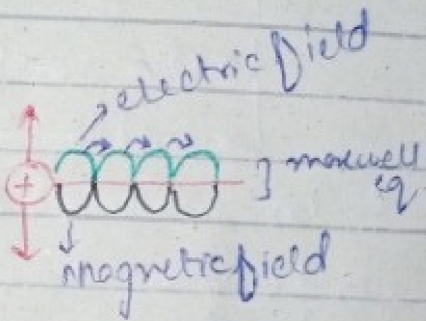
## Types:



### On Basis Of Medium:

1: Mechanical Wave:
↳ Require medium
• Sound wave, Water.

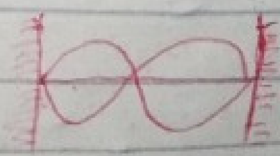
2: Electromagnetic Waves:
↳ No medium Required
• Light, X-ray




### On Basis Of E Transfer:

1: Progressive Waves:
↳ Waves transfer Ⓔ by moving away from source
• Ripples in water

2: Stationary Waves:
↳ Donot transfer Ⓔ
• In strech string
• Organ Pipe



### On Basis Of Oscillation:

1: Transverse Waves:
↳ Particles vibrate perpendicular to the direction of propagation of waves.

# 8: Waves

→ Disturbance in medium.
→ Transfer energy & momentum
→ Donot transfer Matter/mass

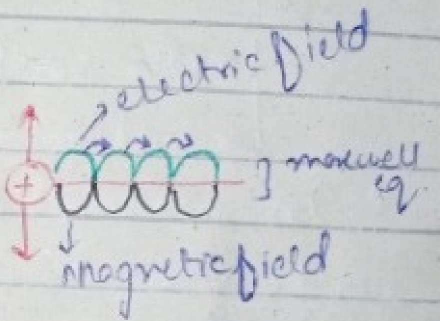
## Types:



### On Basis Of Medium:

1: Mechanical Wave:
↳ Require medium
• Sound wave, Water.

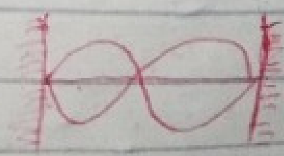
2: Electromagnetic Waves:
↳ No medium Required
• Light, X-ray




### On Basis Of E Transfer:

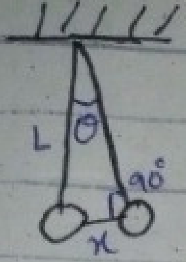
1: Progressive Waves:
↳ Waves transfer Ⓔ by moving away from source
• Ripples in water

2: Stationary Waves:
↳ Donot transfer Ⓔ
• In strech string
• Organ Pipe



### On Basis Of Oscillation:

1: Transverse Waves:
↳ Particles vibrate perpendicular to the direction of propagation of waves.




$$\sin\theta = \frac{P}{H}$$

$$\sin\theta = \frac{x}{L}$$

∴ Opposite of θ it is ⊥ and infront of ⊥ it is H.

$$a = -g\frac{x}{L}$$

$$a = \boxed{-\frac{g}{L}}\, x \quad \rightarrow \text{constant}$$

$a \propto -x$

Time Period:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$T \propto \sqrt{L}$

$T \propto \frac{1}{\sqrt{g}}$

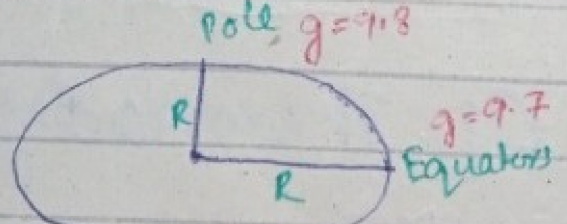
mcq: if $L' = 2L$

$T' = \sqrt{2}\, L$

$T' = (1.4)\, L$

∴ T is independant to mass.

Equator & Pole:




$$g = \frac{GMe}{R^2}$$

Karachi → $g = 9.8$

Murree → $g = 9.7$

$g_m < g_K$

$T_M > T_K$

Intensity:

$I \propto (\text{Amplitude})^2$

$I \propto \frac{1}{(\text{distance})^2}$

∴ If amplitude increases $\boxed{\text{2 times}}$ intensity will increase $\boxed{\text{4 times}}$

# Speed Of Sound:

**For Air** (medium)

$v = \sqrt{\frac{E}{\rho_{med}}}$ → Elastic modulus

**For String**

$v = \sqrt{\frac{T \times L}{m}}$ → Tension

$E = \frac{\sigma}{\varepsilon}$ $\sigma$ → stress, $\varepsilon$ → strain

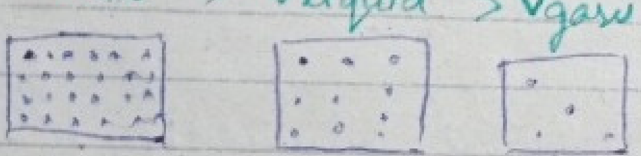
E = solid
(property that resist to change its shape)

$v = \sqrt{\frac{T}{\frac{m}{L}}}$ ∴ $\frac{m}{L} = M$

$v = \sqrt{\frac{T}{M}}$

$$\left(\sqrt{\frac{E}{\rho}}\right)_{solid} > \left(\sqrt{\frac{E}{\rho}}\right)_{liquid} > \left(\sqrt{\frac{E}{\rho}}\right)_{gases}$$

$V_{solid} > V_{liquid} > V_{gases}$

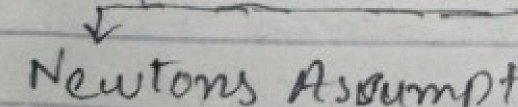


→ Glass → 5500 m/s
→ Iron → 5100 m/s
} 20°C

## Speed Of Sound In Air:

Experimental Value = 332 m/s at 0°C.

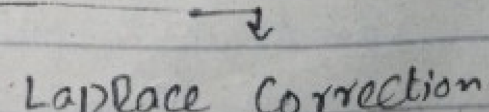
**Newtons Assumption**




→ T = constant
→ isothermal
→ PV = constant

∴ E = P

**Laplace Correction.**




→ Heat = Q = constant
→ Adiabatic process
→ $PV^{\gamma}$ = constant

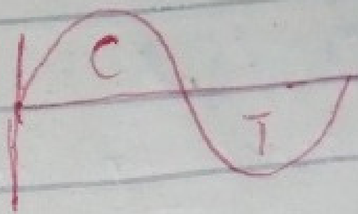
$\gamma$ → monoatomic = 1.6
→ dia = 1.4
→ poly = 1.2

∴ $E = \gamma P$

· Light, String

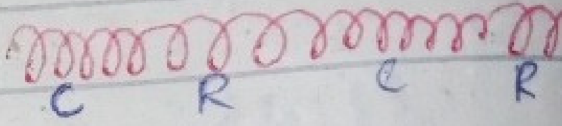
→ Basics of transverse.

↳ Crest & Trough

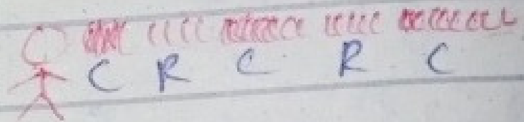



2: Longitudinal Waves:

↳ Particles vibrate parallel.




· Sound waves, Seismic waves




→ Basics:

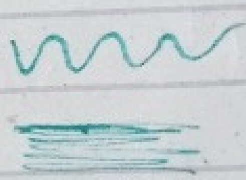
i. ↳ Compression & Rarefaction



Water Waves:

↳ On surface → transverse

↳ Bottom → Longitudinal.



· In water mechanical waves are produced. Transverse waves produces on the surface of water and die out very quickly.

## Relation b/w v, f & λ:

$$v = f\lambda = \frac{\lambda}{T}$$

i: In Same Medium.

$v =$ constant

$f \propto \frac{1}{\lambda}$

ii: When medium Changes

$f =$ constant

$v \propto \lambda$

$\therefore E = P$

$v = \sqrt{\frac{E}{\delta}}$

$v = \sqrt{\frac{P}{\delta}}$

$\boxed{v = 281 m/s}$

$\therefore E = \gamma P$

$v = \sqrt{\frac{E}{\delta}}$

$v = \sqrt{\frac{\gamma P}{\delta}}$

$\boxed{v = 333 m/s}$

Error = 16%

Difference = 52 m/s

# Effect of Speed of Sound:

## i: Pressure:

↳ Speed of sound is independant of to pressure & Frequency.

$v \propto P^0$

If v is speed of sound at 10 atm What is speed at 2 atm?

a: v          b: 2v

c: $\frac{v}{2}$          d: 4v

Speed of sound is directly proportional to:

a: P     b: $P^2$     c: $P^0$

v at 1 atm. Ratio of v for 4 atm?

1 : 1

## ii: Density:

$$v \propto \frac{1}{\sqrt{\rho_{air}}}$$

## iii Wet Air:

↳ fog

$\rho_{air}\downarrow$, $\rho_{water}$

$\uparrow v_{wet}$, $\rho_{air}\downarrow$

$v_{wet\ air} > v_{dry\ air}$

## iv: Dry Air:

$\rho_{air}\uparrow$, $v_{dry}\downarrow$

## v: Wind Velocity: ($v_w$)

$v_{speed} = v_0 + v_w$ $\xrightarrow{\text{sound}}$ $\xrightarrow{\text{air}}$

$\therefore v_0 = 332\ m/s$

$v_{speed} = v_0 - v_w$ $\xrightarrow{\text{sound}}$ $\xleftarrow{\text{air}}$

## vi: Temperature:

$$v = \sqrt{\frac{\gamma R T}{M}}$$

$$v \propto \sqrt{T}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \text{ (Kelvin)}$$

if v is the speed of sound at 27 Kelvin for what value of temperature becomes double?

$v_1 = v$

$v_2 = 2v$

$T = 27K$

$\frac{\cancel{v}}{2\cancel{v}} = \sqrt{\frac{27}{T_2}} \Rightarrow \frac{1}{4} = \frac{27}{T_2}$

$\Rightarrow T_2 = 4 \times 27 = 108K$

Short cut:

$\boxed{T' = n^2 T} \rightarrow$ kelvin

↳ double

↳ $\frac{1}{2}$ (half)

↳ 4 time

$T' = (2)^2 \times 27$

$T' = 4 \times 27$

$\boxed{T' = 108K}$

# Effect Of Temperature:

$V = V_0 + 0.61t \, (C^\circ)$

$\therefore V_0 = 332 m/s$.

$V = 332 + 0.6t$

at $0^\circ C$

$V = 332 + 0.6(0)$

$V = 332 m/s$

at $1C^\circ$:

$V = 332 + 0.61$

On increasing every $1C^\circ$ temperature increases speed by $0.61 m/s$ or $61 cm/s$

# Superposition Principle:

## Interference

Two waves of

- same frequency
- Travelling in same direction
- When they superimposed
- Produces interference.

## Beats

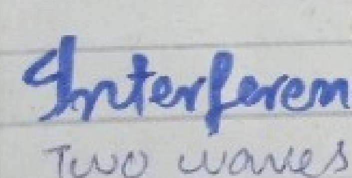
Two waves of

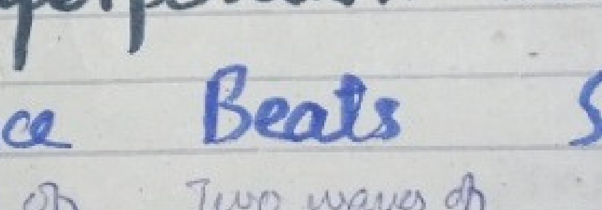
- slightly diff frequency
- Travelling in same direction
- When they superimposed produces beats.

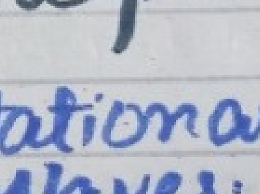
## Stationary Waves:

Two waves of

- Same frequency
- Same amplitude
- Travelling in opposite direction





$7 \times \frac{1}{2} \Rightarrow$ destructive interference.

$B.f = f_1 - f_2$

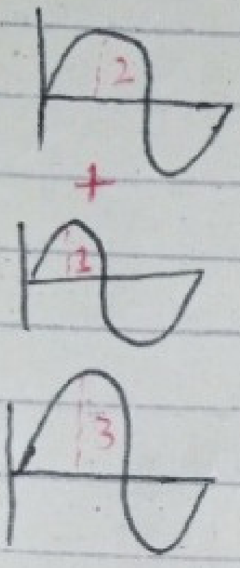
$= 30 - 28$

$B.f = 2 Hz$

# Types of Interference

## Constructive:

When crest falls on crest and Trough falls on Trough.




path difference

$p.d = m\lambda$

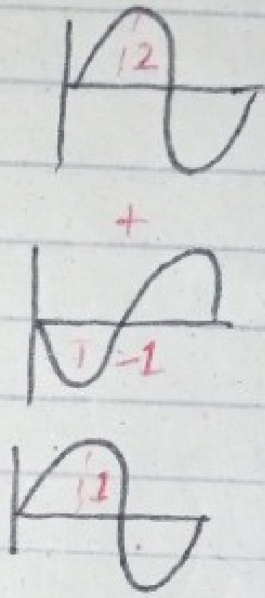
or

$p.d = n\lambda$

$m = 1, 2, 3, 4 \ldots$

## Destructive:

When crest falls on Trough and Trough falls on crest.



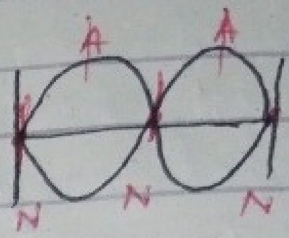

$p.d = (m + \frac{1}{2})\lambda$

$p.d = (2m+1)\frac{\lambda}{2}$

# Stationary Waves:

→ Standing waves
→ Static Waves
→ Non Progressive waves
- → $f, \lambda, A$ same
- → Opposite direction
- → Donot transfer energy
- → $E =$ same
- → $\Delta E = 0$
- → $\Delta P = max$ at Node
- → Strain is maximum at Node

→ $\Delta P$ = minimum at Anti Node
→ Strain minimum at Anti Node.
$\sigma \propto \varepsilon$




Node to Node = $\frac{\lambda}{2}$

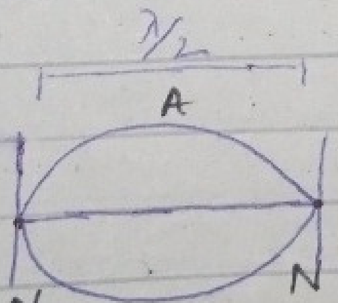
Anti Node to Anti Node = $\frac{\lambda}{2}$

AN−N
N−A.N = $\frac{\lambda}{4}$

Stationary Waves in strech string
→ Transverse stationary waves.
⇒ $v = \sqrt{\frac{T}{m}}$ = constant

Stationary waves in Organ Pipe.
→ Longitudnal stationary waves.
Open at both end
Close at one end.

# Stationary Waves In strech Spring:




N > A

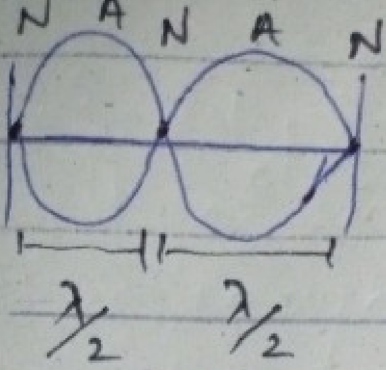
$\therefore v = \sqrt{\frac{T \times L}{m}}$

$v = f\lambda$

$v = f_1 \lambda_1$

$f_1 = \frac{v}{\lambda_1}$ ⇒ $L = \frac{\lambda_1}{2}$ $\lambda_1 = 2L$

$\boxed{f_1 = \frac{v}{2L}}$
→ fundamental frequency
→ ~~Basic~~ 1st harmonic
→ Basic Overtone




L

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

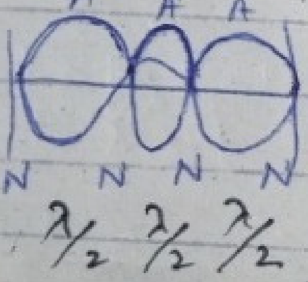
$$L = \frac{2\lambda_2}{2}$$

$$\lambda_2 = \frac{2L}{2}$$

$$f_2 = \frac{V}{\frac{2L}{2}}$$

$$f_2 = 2\frac{V}{2L}$$

$$f_2 = 2 f_1$$ → 2nd harmonic → 1st Overton




$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$L = \frac{3\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{V}{\lambda_3}$$

$$f_3 = \frac{V}{\frac{2L}{3}}$$

$$f_3 = 3\frac{V}{2L}$$

$$f_3 = 3 f_1$$ → 3rd harmonic → 2nd Overton.

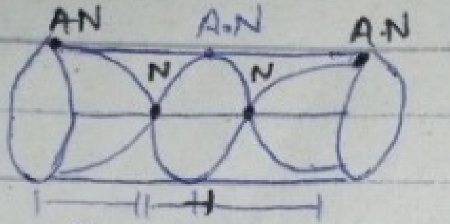
$f_1 = \frac{V}{2L}$, $f_2 = 2f_1$, $f_3 = 3f_1$

$f_4 = 4f_1$, ....., $f_n = nf_1$

$f_n = n\frac{V}{2L}$ $\therefore V = \sqrt{\frac{T \times L}{m}}$

$f_1 \rightarrow$ Smallest frequency
$f_n \rightarrow$ Largest frequency

$\lambda_1 \rightarrow$ Smallest
$\lambda_n \rightarrow$ Largest




$\frac{\lambda}{4}$ $\frac{\lambda}{2}$ $\frac{\lambda}{4}$ → N–N

$L = \frac{\lambda + 2\lambda + \lambda}{4}$

$L = \frac{4\lambda}{4}$ $L = \lambda_2$

$f_2 = \frac{v}{\lambda_2}$

$f_2 = \frac{v}{L}$

$f_2 = \frac{2}{2}\left(\frac{v}{L}\right)$

$f_2 = 2f_1$

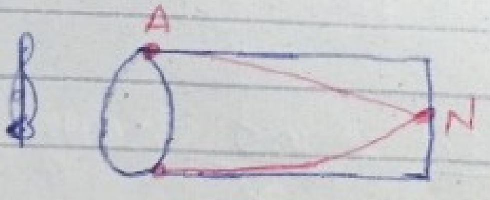
$f_2 = 2f_1$, $f_3 = 3f_1$

$f_4 = 4f_1$, $f_n = nf_1$

∴ All harmonics are present in "both end open organ pipe".

∴ Ritcher harmonics

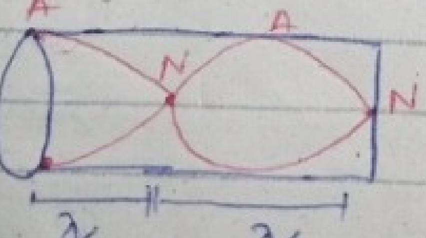
# One End Close: (A=N)




$L = \frac{\lambda}{4} \Rightarrow \lambda_1 = 4L$

$f_1 = \frac{v}{\lambda_1}$

$f_1 = \frac{v}{4L}$ → Fundamental f → 1st harmonic → Basic Overton




$\lambda/4$ $\lambda/2$

$L = \frac{\lambda}{4} + \frac{\lambda}{2} = \frac{\lambda + 2\lambda}{4}$

$L = \frac{3\lambda_2}{4}$ $\lambda_2 = \frac{4L}{3}$

$f_2 = \frac{v}{\lambda_2}$

$f_2 = \frac{v}{\frac{4L}{3}}$

$f_2 = 3\frac{v}{4L}$ → 3rd harmonic

$f_2 = 3f_1$ → 1st Overton

$v = 40\,m/s$

$L = 2m$

$f_1 = ?\quad \frac{v}{2L} = \frac{40}{4} = 10$

$f_2 = 2(10) = 20$

$f_3 = 3(10) = 30$

$f_4 = 4(10) = 40)$

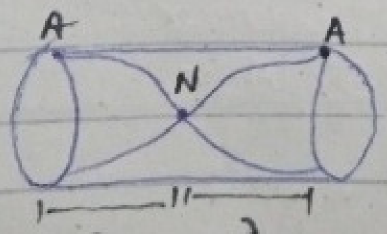
$\lambda_n = \frac{\lambda_1}{n}$ or $\lambda_n = \frac{2L}{n}$

$\lambda_1 = \frac{2L}{1}$ , $\lambda_2 = \frac{2L}{2}$

$\lambda_3 = \frac{2L}{3}$ , $\lambda_4 = \frac{2L}{4}$

# Organ Pipe:

## Open At Both end: (A > N)




$\lambda/4 \quad \lambda/4$

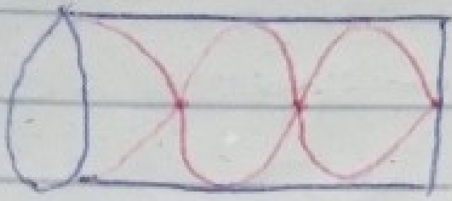
$f_1 = \frac{v}{\lambda_1}$

$f_1 = \frac{v}{2L}$ → Fundamental f → 1st harmonic → Basic Overtone

$L = \frac{\lambda}{4} + \frac{\lambda}{4}$

$L = \frac{\lambda}{2}$

$\lambda = 2L$



$\lambda_3 = \frac{4L}{5}$

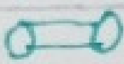
$L = \frac{5\lambda_3}{4}$

$f_3 = \frac{v}{\lambda_3} \Rightarrow \frac{v}{\frac{4L}{5}} \Rightarrow f_3 = 5\left(\frac{v}{4L}\right)$

$f_3 = 5f_1$ → 5th harmonic
→ 2nd Overtor

$f_4 = 7f_1$ → 7th H.M
→ 3rd Overton

$f_5 = 9f_1$

H.M  > H.M 

$f_n = (2n-1)f_1$

odd harmonics only

---

$F_o = \frac{v}{2L}$ → open , $F_c = \frac{v}{4L}$ → close

$F_c = \frac{v}{2 \times 2L}$

$\boxed{F_c = \frac{F_o}{2}}$

$F_o = 100Hz$

$F_c = ?$

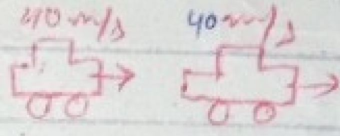
$F_c = \frac{F_o}{2} = \frac{100}{2} = 50Hz$

# Doppler's Effect:

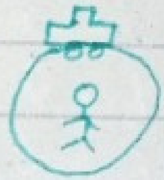
The apparent change in frequency due to relative motion of source (S) and Listener (L).

## Limitations:

→ Valid for all types of waves.
→ Motion must be relative.




→ If distance is constant, Doppler effect is not valid



$f \rightarrow$ Real frequency
$f' \rightarrow$ Apparent frequency
(S) → Source, (L) → Listner
(a) → Speed of ~~Listener~~ Source
(b) → Speed of Listner.

**Case 1:** When Source (S) moves & Listner (L) at Rest:

i: Source moves towards Listner: ((S) → (L))

$$f' = \left(\frac{v}{v-a}\right)f \qquad f' > f$$

ii: (S) moves away from (L): (← (S) (L))

$$f' = \left(\frac{v}{v+a}\right)f \qquad f' < f$$

**Case 2:** When (S) at rest (L) moves:

i: (S) ← (L) : Listner moves towards Source.

$$f' = \left(\frac{v+b}{v}\right)f \qquad f' > f$$

ii: (S) (L) → : Listner moves away from Source.

$$f' = \left(\frac{v-b}{v}\right)f \qquad f' < f$$

Case 3: When both moves

①: Source $\rightarrow$ $\leftarrow$ Listner

$$f' = \left(\frac{v+b}{v-a}\right) f$$

②: $\leftarrow$ (S) (L) $\rightarrow$

$$f' = \frac{(v-b)}{v+a} f$$

**MCQ:**

If a source is moved with speed $\frac{4}{5}v$. Receeding the stationary (L). What $\frac{f'}{f} = ?$

$$f' = \left(\frac{v}{v+a}\right) f \Rightarrow \frac{f'}{f} = \frac{v}{v+a} \Rightarrow \frac{f'}{f} = \frac{v}{v+\frac{4}{5}v}$$

$$\frac{f'}{f} = \frac{v}{\frac{5v+4v}{5}} \Rightarrow \frac{5\cancel{v}}{9\cancel{v}} \Rightarrow \boxed{\frac{f'}{f} = \frac{5}{9}}$$

**MCQ:**

If (L) is moving towards stationay (S) with 10 m/s. The (S) producing f of 332 Hz. What is apparent $f'$?

$$f' = \left(\frac{v+b}{v}\right) f$$

if v is not given, it will be 332 m/s.

$$= \left(\frac{332+10}{\cancel{332}}\right) \cancel{332}$$

$$= 332 + 10$$

$$f' = 342\,Hz$$

## Applications:

i: Radar: Radio waves

ii: Sonar: Sound waves (water)

iii: Speed Trap: Micro waves (speed monitoring)

iv: Motion of stars: → Star moving towards us show blue shift:

→ Star moving away from us show Red shift.

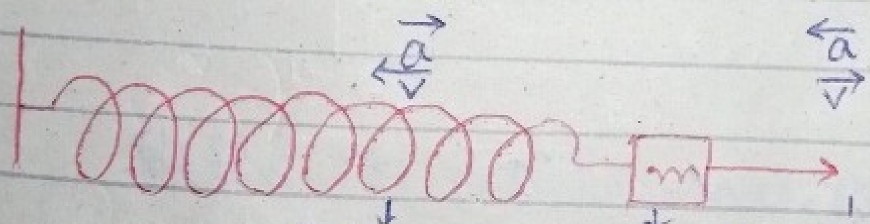
VIBGYOR

# Simple Harmonic Motion:

$\vec{a}$ is directly proportional to $\vec{x}$ and always directed towards mean position.

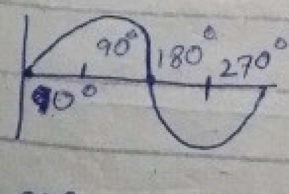
$$\vec{a} \propto -\vec{x}$$

$$\vec{F} \propto -\vec{x}$$

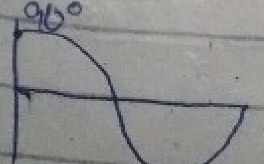



| | Extreme P | Mean P | Extreme Position |
|---|---|---|---|
| ∴ When K.E is maximum P.E is 0. | $x = x_0$ | $x = 0$ | $x = x_0$ |
| ∴ When $\vec{v}$ is max $\vec{a}$ is 0. | $V = 0$ | $V = max$ | $V = 0$ |
| | $a = max$ | $a = 0$ | $a = max$ |
| | $K.E = 0$ | $K.E = max$ | $K.E = 0$ |
| | $P.E = max$ | $P.E = 0$ | $P.E = max.$ |

$x \sin\theta$




$x \cos\theta$



For what value of '$x$' $K.E = P.E$ :

$P.E = K.E$

$\frac{1}{\cancel{2}}Kx^2 = \frac{1}{\cancel{2}}\cancel{K}(x_0^2 - x^2)$

$x^2 = x_0^2 - x^2$

$2x^2 = x_0^2$

$x^2 = \frac{x_0^2}{2}$

$x = \frac{x_0}{\sqrt{2}}$

$x = (0.70)\, x_0$

$x = (70\%)\, x_0$

For what value of $\theta$ $K.E = P.E$ ?

$K.E = P.E$

$x_0 \cos\theta = x_0 \sin\theta$

$\cos\theta = \sin\theta$

$\boxed{\theta = 45^\circ}$

* In 1 vibration, $K.E = P.E \longrightarrow$ 4 times
* In 1 vibration, $K.E$ & $P.E \longrightarrow$ Maximum $\rightarrow$ twice
- In 1 vibration displacement. $x_0 = 0$
- In 1 vibration distance $x = 4$ times

# Spring Constant:

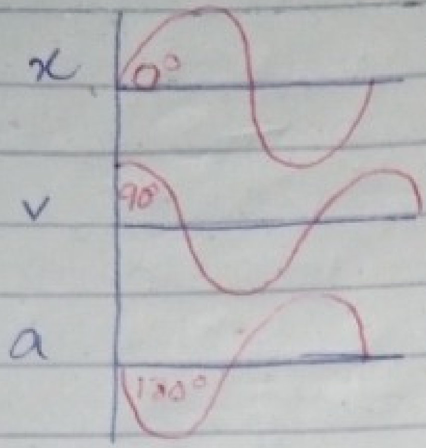
$F \propto x$ (Hooks law)

$\boxed{F = Kx}$

$K = \frac{F}{x}$

$K = \frac{N}{m} = \frac{Kg\,m\,s^{-2}}{m} = K$

$K = Kg\, s^{-2}$

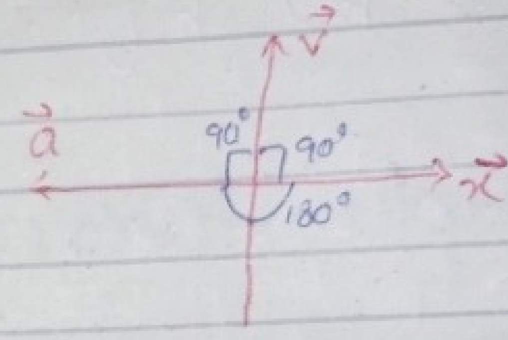



$x = x_0 \sin\theta$

$v = v_0 \cos\theta$

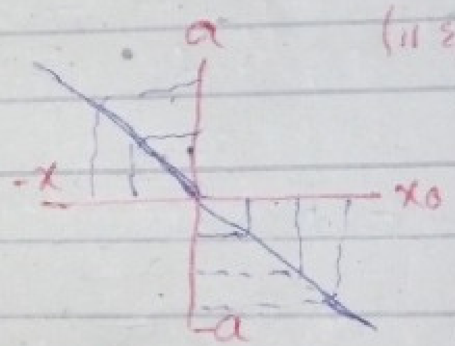
$a = -a_0 \sin\theta$

$\theta$ b/w $\vec{x}$ & $\vec{v}$ = 90°
$\theta$ b/w $\vec{v}$ & $\vec{a}$ = 90°
$\theta$ b/w $\vec{x}$ & $\vec{a}$ = 180°



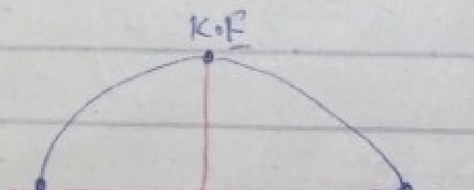

$a \propto -x$

| | |
|---|---|
| $x = 1, a = -1$ | $x = -1, a = 1$ |
| $x = 2, a = -2$ | $x = -2, a = 2$ |
| $x = 3, a = -3$ | $x = -3, a = 3$ |




(II & IV quad

# Energy Conservation in SHM:

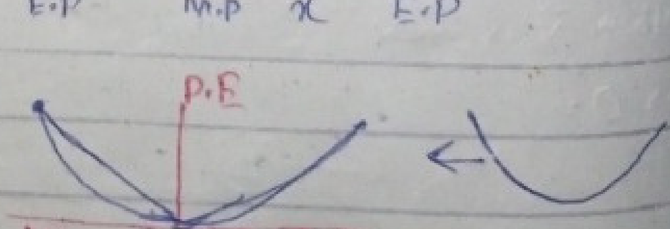


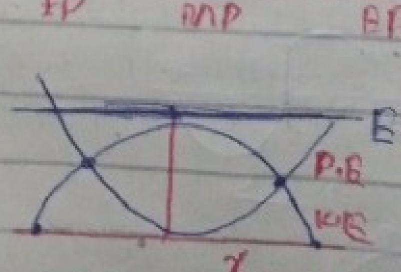

E = conserved

$E = \frac{1}{2} K x_0^2$

$K.E = \frac{1}{2} k (x_0^2 - x^2)$

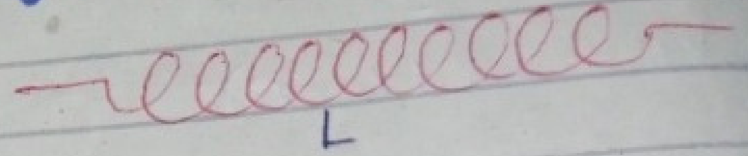
$P.E = \frac{1}{2} K x^2$



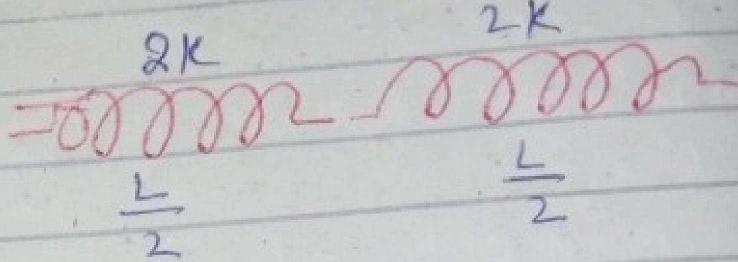




Energy is parallel to x.

# Dependence Of K:




i: $K \propto \frac{1}{\text{Length}}$




ii: $K \propto \frac{1}{\text{Temperature}}$

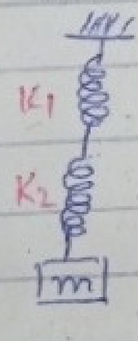
→ $L' = \frac{2}{3}L$, $K' = \frac{3}{2}K$

∴ Length ↓ n time, K will ↑ n time.

→ $L' = \frac{5}{4}L$, $K' = \frac{4}{5}K$

# Combination Of K:

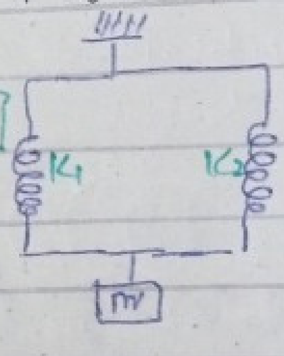
Series:




$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_{eq}} = \frac{K_1 K_2}{K_1 + K_2}$$

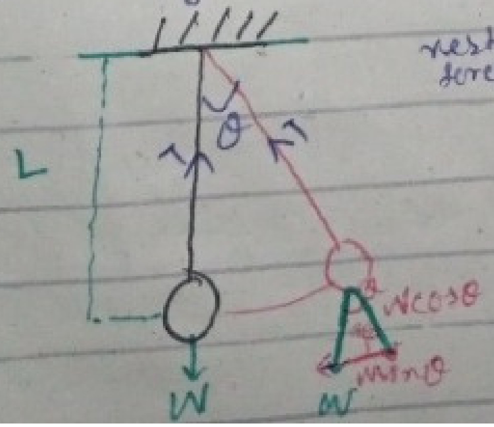
Parrallel




$\boxed{K_{eq} = K_1 + K_2}$

# Simple Pendulum:

↳ A metallic bob is attach with one end of string called pendulum.




$T = W\cos\theta$ balance each other

restoring force ← $F_r = -W\sin\theta$

$ma = -mg\sin\theta$

$\boxed{a = -g\sin\theta}$

For smaller Amplitude

$\sin\theta \approx \theta$